



Two planes on merging routes are:
--different distances from the intersection
--traveling at different speeds.

SMART SKIES™ FLYBY MATH™

Distance-Rate-Time Problems in Air Traffic Control for Grades 5 - 9

AIR TRAFFIC CONTROL PROBLEM 4

Teacher Guide with Answer Sheets

Overview of Air Traffic Control Problem 4

In this Air Traffic Control (ATC) Problem, students will determine if two airplanes traveling on different merging routes will conflict with (meet) one another at the intersection of their flight routes.

The airplanes are each a **different distance from the point of intersection**. The airplanes are traveling at **different constant (fixed) speeds**, and the airplane closer to the intersection is traveling more slowly.

This scenario is the most complicated of the merging cases. It builds upon the simpler case set forth in ATC Problem 3 in which the planes are also traveling at different speeds. In ATC Problem 3, each plane is the same distance from the intersection. In the current problem, one of the planes has a “headstart.”

Objectives

Students will determine the following:

- If two planes are traveling at different constant (fixed) speeds on two different routes and the planes are each a different distance from the point where the two routes come together, students must know those values in order to determine the separation distance between the planes at the intersection.

Materials

Student handouts:

ATC Problem 4 Student Workbook
ATC Problem 4 Assessment Package (optional)

The student handouts are available on the *FlyBy Math™* website:

<http://smarts skies.arc.nasa.gov>



Introducing Your Students to the ATC Problem

You may want to show the FlyBy Math™ video clips to introduce your students to the air traffic control system.

(For more detail, see the FlyBy Math™ Educator Guide.)

Student Workbook

*It may help your students to think of the plane speeds in inches per second:
6 inches/second
4 inches/second*

Materials for the experiment:

- sidewalk chalk or masking tape or cashier's tape or a knotted rope
- measuring tape or ruler
- marking pens (optional)
- 1 stopwatch or 1 watch with a sweep second hand or 1 digital watch that indicates seconds
- pencils
- signs (available on the *FlyBy Math™* website) identifying pilots, controllers, and NASA scientists
- clipboard (optional)

To help your students understand the problem, you can ask them to consider this related problem that is set in a more familiar context:

Two students, Ana and Alex, plan to meet at the movies. Ana lives 20 blocks from the theater. Alex lives 16 blocks from the theater. Ana and Alex will each leave their homes at the same time and walk at different constant (fixed) speeds. Alex walks more slowly than Ana.

You can ask your students these questions:

Who will arrive at the theater first, Ana or Alex? Why?

How many blocks apart will they be when the first person arrives at the theater?

The answers are not intuitive and students should realize that they must know the speeds and analyze the situation to determine the answers.

For a detailed description of the Student Workbook features found in each ATC Problem, see the *FlyBy Math™* Educator Guide.

The following section addresses *special features* of the ATC Problem 4 Workbook.

Read the Problem

The speed of one airplane is $\frac{1}{2}$ foot/second. That airplane is 20 feet from the point of intersection. The speed of the other airplane is $\frac{1}{3}$ foot/second. That airplane is 16 feet from the point of intersection.

Note: These speeds and distances were chosen to reflect the classroom experiment that the students will conduct and are not related to real-world parameters.



As a problem enhancement, you may want to ask your students to solve the problem using real-world data.

In a real-world scenario, one plane might be traveling at 400 nautical miles per hour. That plane might be 40 nautical miles from the point of intersection. The other plane might be traveling at 320 nautical miles per hour. That plane might be 36 nautical miles from the intersection.

An international nautical mile is 1,852 meters.

A nautical mile per hour is called a “knot”.

Set Up and Do the Experiment

A complete description of this section is contained in the *FlyBy Math™* Educator Guide.

Do the Calculations

Each of the six calculation methods is described in the *FlyBy Math™* Educator Guide.

One method, Graph Two Linear Equations, is described in greater detail below.

--Graph Two Linear Equations

Caution: Students may confuse the path of a plane with the graph of the plane's distance traveled as a function of time. In particular, students may confuse the intersection of the graphs with a collision of the planes. At the time the graphs intersect, each plane is 8 feet from the intersection. Since the planes are traveling on separate routes, the planes do not intersect at the point where their graphs intersect.

Analyze Your Results

As part of the Analysis, you may also want to ask your students to create a similar problem in a different setting. They have already considered a problem in which two students walk from their respective homes to a movie theater. (See the *Introducing Your Students to the ATC Problem* section of this document.)

Now, you might suggest they consider two cars traveling in parallel lanes on the same road, with the two lanes merging into one lane. Each car is traveling at a different constant (fixed) speed. Each car is a different distance from the merge. The car closer to the intersection is traveling more slowly.

Note: To be consistent with the airspace scenarios, it is important that for each problem created by you or your students, you choose a fixed (constant) speed for each vehicle or person. (For example, a rocket launch scenario would *not* be appropriate because a launched rocket typically accelerates and therefore its speed is not constant.)

**Extension**

The extension introduces a separation requirement at the point where the routes intersect. For safety reasons, when the first plane reaches the intersection, the planes must be separated by a distance greater than or equal to a given standard separation distance. If their separation is less than this standard, a separation violation will occur.

Students are asked to review their calculations to determine the separation distance between the planes at the intersection of the routes. They are then asked whether that distance meets the separation requirement.

Answers and Explanations

The first part of this section summarizes the answers to the key questions posed in the ATC Problem. The remainder of this section is organized by activity and includes graphs, diagrams, and answers to individual questions, as well as discussions of the general problems posed in the analysis activity and the posttest.

Answer Summary

The speed of Flight WAL27 is $\frac{1}{2}$ foot/second, so the plane travels $\frac{1}{2}$ foot in 1 second.

The speed of Flight NAL63 is $\frac{1}{3}$ foot/second, so the plane travels $\frac{1}{3}$ foot in 1 second.

Flight WAL27 is 20 feet from the point of intersection.

Flight NAL63 is 16 feet from the point of intersection.

The planes are traveling at different constant (fixed) speeds from different starting distances. The plane closer to the intersection is traveling more slowly than the other plane. So it is not intuitive as to whether there will be a conflict at the intersection.

After analyzing the situation, students can determine that Flight WAL27 will arrive first at the intersection, even though the flight started further from the intersection. This is because Flight WAL27 is going 50% faster than Flight NAL63, but Flight WAL27 starts only 25% further away than Flight NAL63.

To obtain the percentages in the previous paragraph:

- Compare the difference in speed, $\frac{1}{6}$ foot per second, with the speed of Flight NAL63.
- Compare the initial separation distance, 4 feet, with the starting distance of Flight NAL63.

In particular:

- It will take 40 seconds for Flight WAL27 to travel 20 feet to the point where the routes come together.
- It will take 48 seconds for Flight NAL63 to travel 16 feet to the point where the routes come together.



Answers by Activity

- At 40 seconds, when Flight WAL27 arrives at the intersection, Flight NAL63 will be approximately 2.7 feet from the intersection. So the planes will be approximately 2.7 feet apart when the first plane arrives at the intersection.

Answers are provided for all worksheets including the Pretest, Student Workbook, and Posttest. Please see the following pages.

Note: Answers are given only for the numbered activity steps that require students to provide a numerical or verbal response. For example, if a step requires a student to trace or circle a portion of a diagram, that step is *not* included in the Answers.



ANSWERS BY ACTIVITY

Pretest

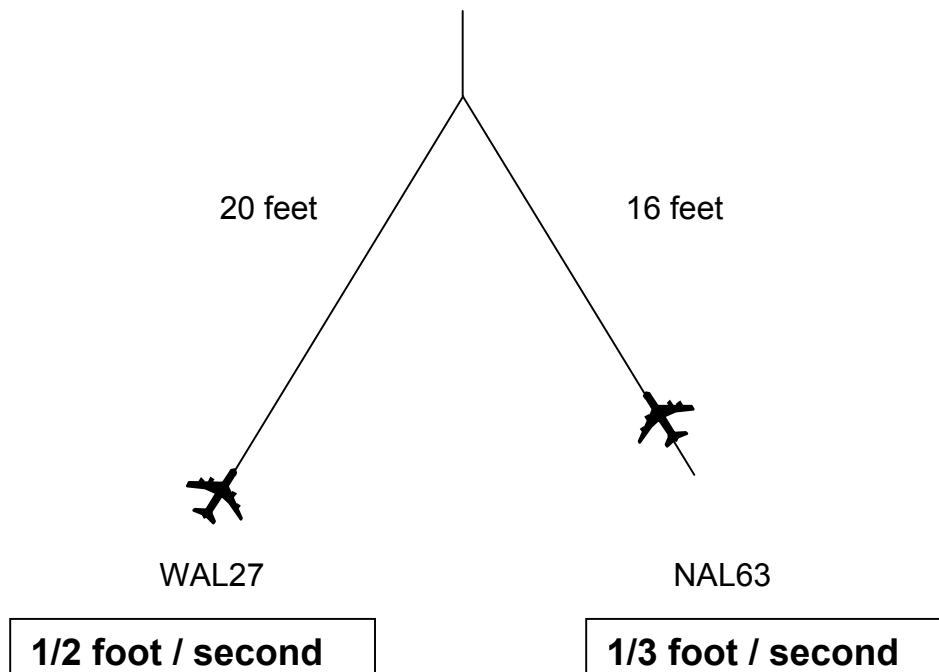
1. Do you think the planes will arrive at the same time
at the point where the two routes intersect?

No

Why or why not? **Flight WAL27 will arrive first because it is traveling 50% faster than Flight NAL63, but Flight WAL27 starts only 25% further away than Flight NAL63.**

2. If not, how many feet apart do you think the planes will be
when the first plane reaches the point where the routes intersect?

Approximately 2.7 feet

**Student Workbook: Read the Problem**

2. How far does WAL27 travel in 1 second? **$\frac{1}{2}$ foot**
3. How far does WAL27 travel in 10 seconds? **5 feet**
5. How far does NAL63 travel in 1 second? **$\frac{1}{3}$ foot**
6. How far does NAL63 travel in 10 seconds? **Approximately 3.3 feet**



Student Workbook: Do the Calculations—Count Feet and Seconds

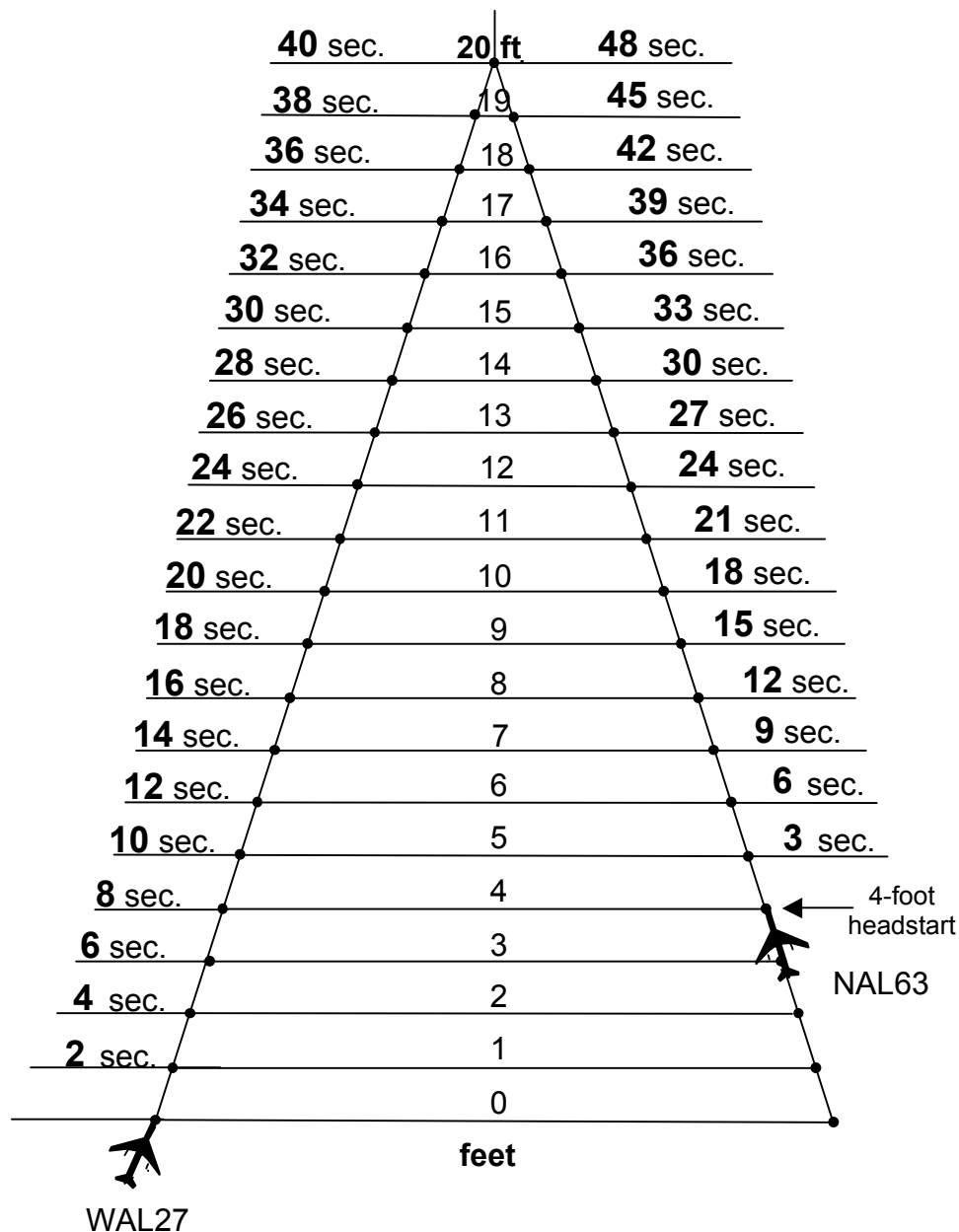
Discussion: Flight WAL27 travels 1 foot in 2 seconds. Count by 2s to fill in the seconds in the blanks along the jet route.

Flight NAL63 travels 1 foot in 3 seconds. Count by 3s to fill in the seconds in the blanks along the jet route.

Flight WAL27 will travel 20 feet in 40 seconds. (Students can also multiply 2 seconds per foot by 20 feet to obtain 40 seconds.)

Flight NAL63 will travel 16 feet in 48 seconds. (Students can also multiply 3 seconds per foot by 16 feet to obtain 48 seconds.)

So Flight WAL27 arrives at the intersection 8 seconds ahead of Flight NAL63.



**Student Workbook: Do the Calculations—Count Feet and Seconds (cont.)**

7. How many seconds did it take each plane to arrive at the point where the routes intersect?

WAL27 40 seconds

NAL63 48 seconds

8. Did the planes meet at the point where
the two routes intersect?

No

9. If No, which plane arrived first?

WAL27

10. How many seconds did it take this plane to travel
to the point where the two routes intersect?

40 seconds

11. At that time, how far away was the other airplane? (Hint: At that time, how many
feet had the second plane traveled? How many feet was it from the intersection?)

Flight NAL63 was approximately 2.7 feet away.

**After 40 seconds, Flight NAL63 had traveled approximately 13.3 feet
along its 16-foot route. So NAL63 was approximately 2.7 feet from the
intersection. So the planes do not meet.**

12. If you think two planes will meet, what would you tell the air traffic controller
to do to avoid a collision?

Change the speed or change the route of one of the planes.

13. You moved along each jet route, one foot at a time, to find the number of seconds it took each
plane to travel to the point where the routes meet. Can you think of a faster way to find the number
of seconds? If so, describe the faster way.

**For Flight WAL27, multiply 2 seconds per foot by 20 feet to obtain 40
seconds. For Flight NAL63, multiply 3 seconds per foot by 16 feet to
obtain 48 seconds.**



Student Workbook: Do the Calculations—Draw Blocks

Discussion:

Flight WAL27 travels 1 foot in 2 seconds. So in 10 seconds, Flight WAL27 will go 5 feet.

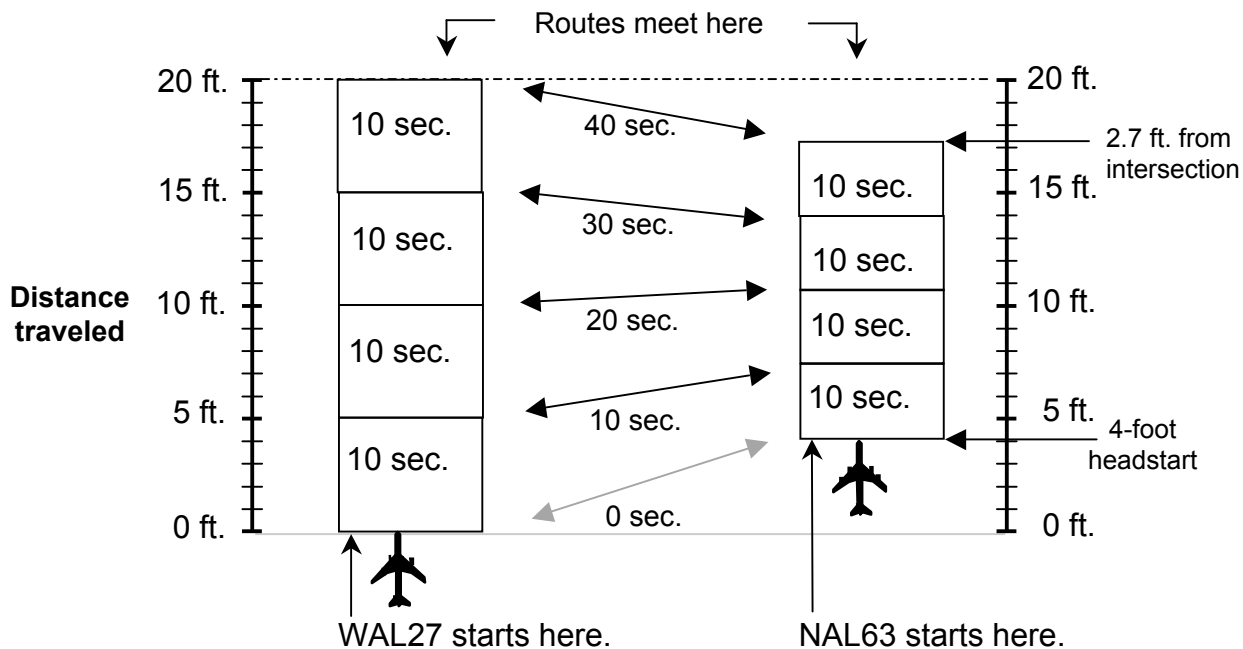
Flight NAL63 travels 1 foot in 3 seconds. So in 10 seconds, Flight NAL63 will go $3\frac{1}{3}$ feet (approximately 3.3 feet).

The following diagram shows a stack of 10-second blocks for each plane.

In the stack corresponding to Flight WAL27, each block represents 5 feet.

In the stack corresponding to Flight NAL63, each block represents approximately 3.3 feet.

As the blocks for Flight WAL27 are stacked, it will become clear that the flight arrives at the intersection in 40 seconds. At this time, Flight NAL63 has traveled approximately 13.3 feet and is approximately 2.7 feet from the intersection. So a conflict will not occur.



1. Flight WAL27 takes 10 seconds to travel ...

5 feet

2. Flight NAL63 takes 10 seconds to travel ...

Approximately 3.3 feet

**Student Workbook: Do the Calculations—Draw Blocks (cont.)**

16. Did the planes meet at the point where the two routes intersect?

No

17. If No, which plane arrived first?

WAL27

18. How many seconds did it take this plane to travel to the point where the two routes intersect?

40 seconds

19. At that time, how far away was the other airplane? (Hint: At that time, how many feet had the second plane traveled? How many feet was it from the intersection?)

Flight NAL63 was approximately 2.7 feet away.

After 40 seconds, Flight NAL63 had traveled approximately 13.3 feet along its 16-foot route. So NAL63 was approximately 2.7 feet from the intersection. So the planes do not meet.

20. If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?

Change the speed or change the route of one of the planes.



Student Workbook: Do the Calculations—Plot Points on Lines

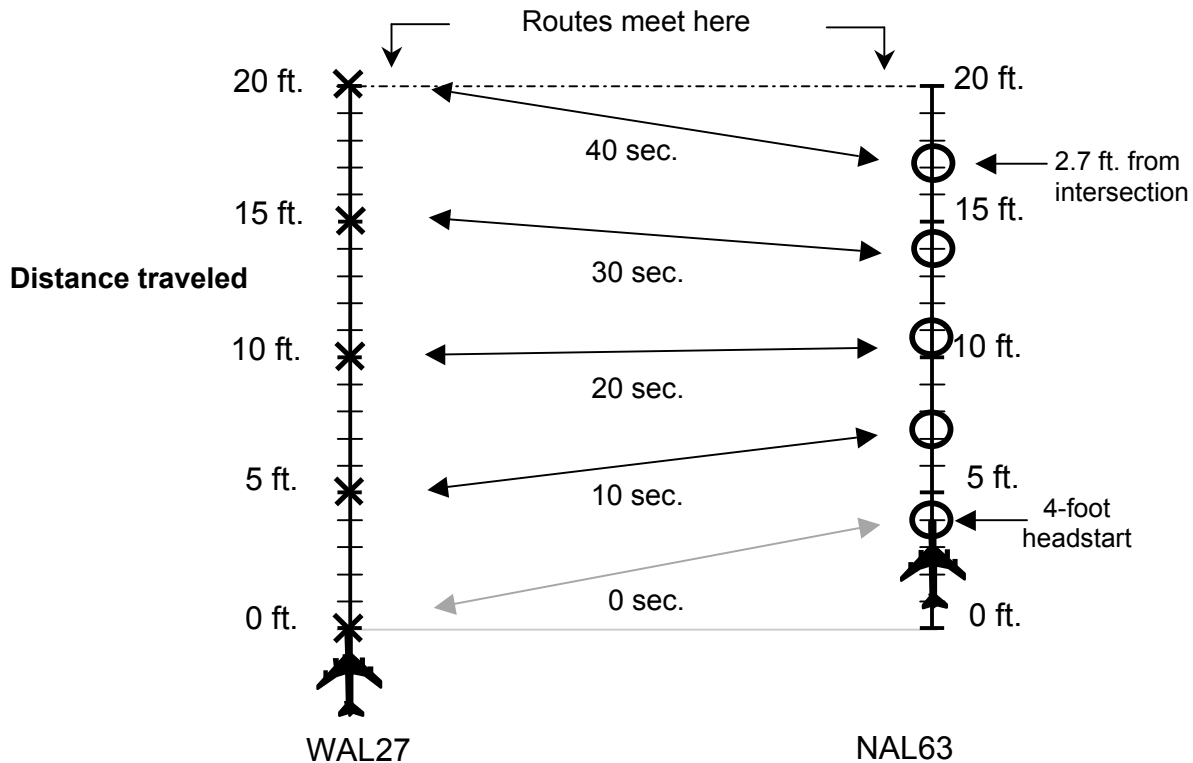
Discussion:

Flight WAL27 travels 1 foot in 2 seconds. So in 10 seconds, Flight WAL27 will go 5 feet.

Flight NAL63 travels 1 foot in 3 seconds. So in 10 seconds, Flight NAL63 will go $3\frac{1}{3}$ feet (approximately 3.3 feet).

The following diagram shows the position of each plane at 10-second intervals.

As the points for Flight WAL27 are plotted, it will become clear that the flight arrives at the intersection in 40 seconds. At this time, Flight NAL63 has traveled approximately 13.3 feet and is approximately 2.7 feet from the intersection. So a conflict will not occur.



14. Did the planes meet at the point where the two routes intersect?

No

15. If No, which plane arrived first?

WAL27



Student Workbook: Do the Calculations—Plot Points on Lines (cont.)

16. How many seconds did it take this plane to travel
to the point where the two routes intersect?

40 seconds

17. At that time, how far away was the other airplane? (Hint: At that time, how many feet had the second plane traveled? How many feet was it from the intersection?)

Flight NAL63 was approximately 2.7 feet away.

After 40 seconds, Flight NAL63 had traveled approximately 13.3 feet along its 16-foot route. So NAL63 was approximately 2.7 feet from the intersection. So the planes do not meet.

18. If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?

Change the speed or change the route of one of the planes.


Student Workbook: Do the Calculations—Plot Points on a Grid

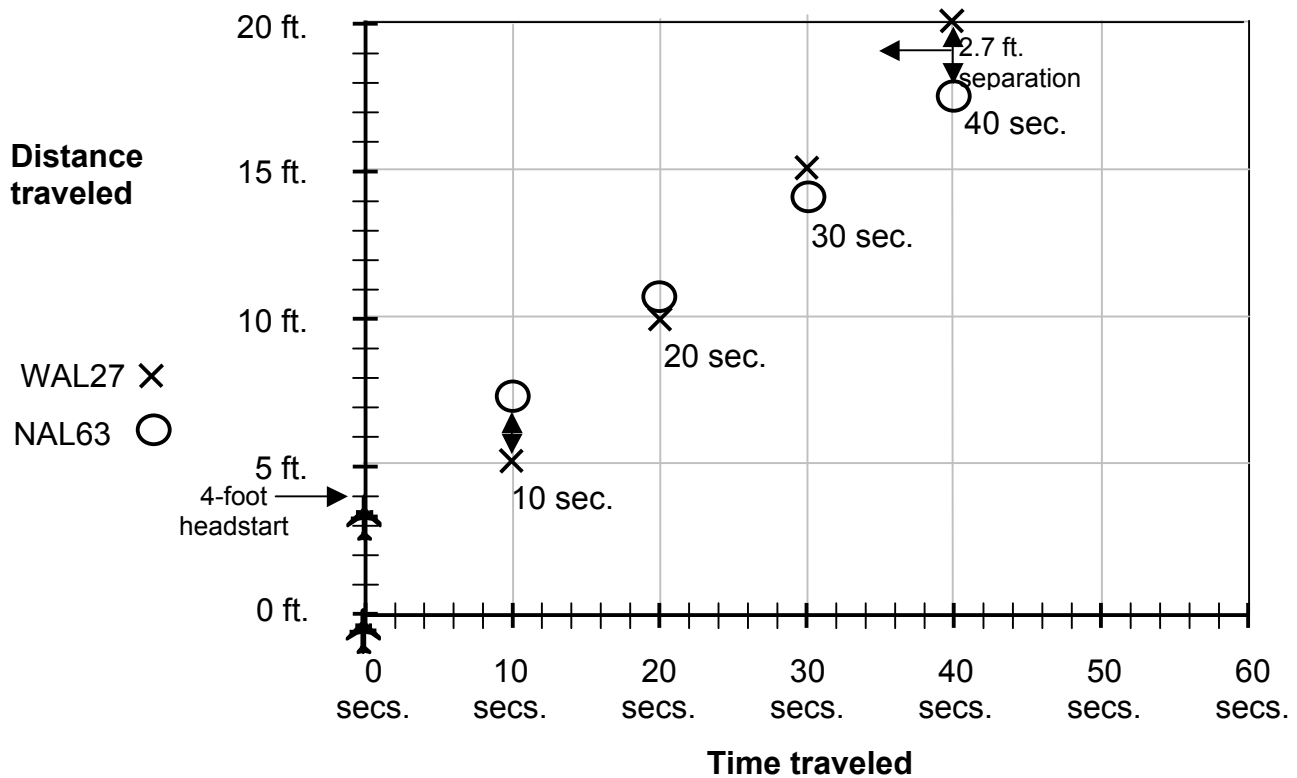
Discussion:

Flight WAL27 travels 1 foot in 2 seconds. So in 10 seconds, Flight WAL27 will go 5 feet.

Flight NAL63 travels 1 foot in 3 seconds. So in 10 seconds, Flight NAL63 will go $3\frac{1}{3}$ feet (approximately 3.3 feet).

The following graph shows the position of each plane at 10-second intervals.

As the points for Flight WAL27 are plotted, it will become clear that the flight arrives at the intersection in 40 seconds. At this time, Flight NAL63 has traveled approximately 13.3 feet and is approximately 2.7 feet from the intersection. So a conflict will not occur.



13. Did the planes meet at the point where the two routes intersect?

No

14. If No, which plane arrived first?

WAL27



Student Workbook: Do the Calculations—Plot Points on a Grid (cont.)

15. How many seconds did it take this plane to travel
to the point where the two routes intersect?

40 seconds

16. At that time, how far away was the other airplane? (Hint: At that time, how many feet had the second plane traveled? How many feet was it from the intersection?)

Flight NAL63 was approximately 2.7 feet away.

After 40 seconds, Flight NAL63 had traveled approximately 13.3 feet along its 16-foot route. So NAL63 was approximately 2.7 feet from the intersection. So the planes do not meet.

17. If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?

Change the speed or change the route of one of the planes.

**Student Workbook: Do the Calculations—Use a Formula**

Discussion: Flight WAL27 arrives at the intersection first, in 40 seconds.

Flight NAL63 arrives at the intersection 20 seconds later, at 60 seconds.

Since the times are different, a conflict will not occur.

1. -In 4 seconds, Flight WAL27 travels **0.5 feet/second** × **4 seconds** = **2.0** feet.

-In 5 seconds, Flight WAL27 travels **0.5 feet/second** × **5 seconds** = **2.5** feet.

2. How could you use multiplication to find the distance Flight WAL27 travels in 14 seconds? **Multiply 0.5 feet/second by 14 seconds.**

3. Use the formula

$$d = r \cdot t$$

to answer this question.

How many feet does Flight WAL27 travel in 20 seconds? **10 feet**

4. Use the formula

$$d = r \cdot t$$

to answer this question.

How many feet does Flight NAL63 travel in 20 seconds? **Approx. 6.7 feet**

**Student Workbook: Do the Calculations—Use a Formula (cont.)**

5. Use the formula

$$t = \frac{d}{r}$$

to find the number of seconds for WAL27 to travel 20 feet to the point where the routes meet

$$t = \frac{20 \text{ feet}}{0.5 \text{ feet per second}} = \underline{40} \text{ seconds}$$

6. Use the same formula to find the number of seconds for NAL63 to travel 16 feet to the point where the routes meet.

$$t = \frac{16 \text{ feet}}{1/3 \text{ foot per second}} = \underline{48} \text{ seconds}$$

7. Will the planes meet at the point where the two routes intersect?

No

8. If No, which plane will arrive first?

WAL27

**Student Workbook: Do the Calculations—Use a Formula (cont.)**

9. How many seconds will it take this plane to travel
to the point where the two routes intersect?

40 seconds

10. At that time, how far away is the other airplane? (Hint: At that time, how many feet has the second plane traveled? How many feet is it from the intersection?)

Flight NAL63 is approximately 2.7 feet away.

After 40 seconds, Flight NAL63 has traveled approximately 13.3 feet along its 16-foot route. So NAL63 is approximately 2.7 feet from the intersection. So the planes do not meet.

11. If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?

Change the speed or change the route of one of the planes.



Student Workbook: Do the Calculations—Graph Linear Equations

Discussion: Flight WAL27 arrives at the intersection first, in 40 seconds. On the line corresponding to Flight NAL63, locate the point where the time is 40 seconds. That point corresponds to a distance approximately 2.7 feet from the intersection. So the planes do not meet at the intersection.

1. Fill in the table for **WAL27**.

2. Fill in the table for **NAL63**.

$$d = (1/2)t$$

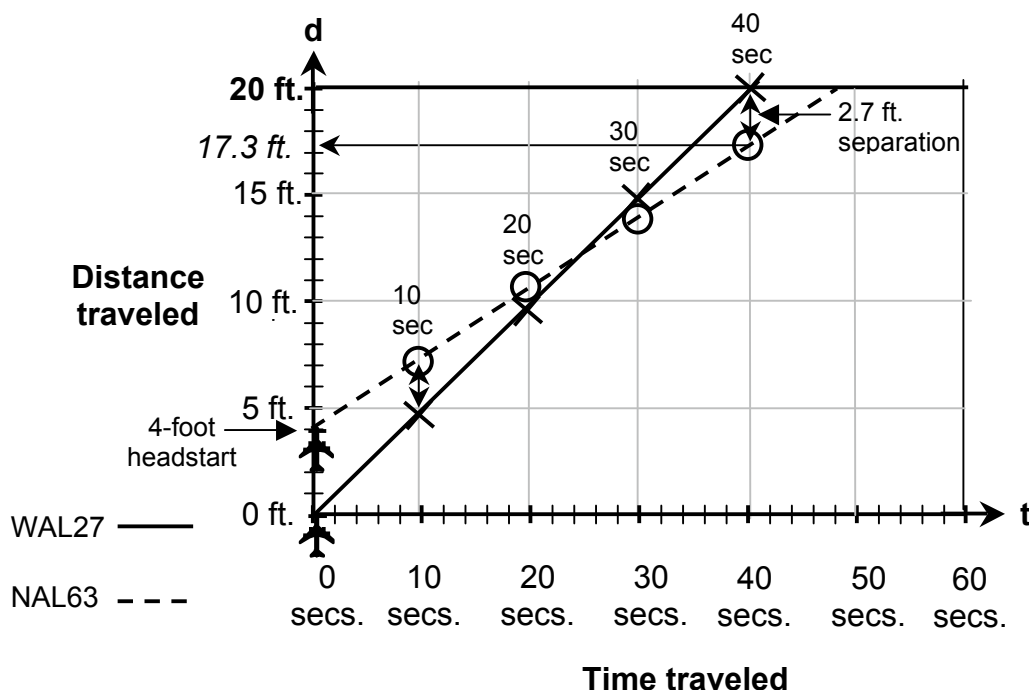
t seconds	d feet
0	0
10	5
20	10
30	15
40	20

$$d = (1/3)t + 4$$

t seconds	d feet
0	4
10	7.3
20	10.7
30	14
40	17.3

3. Use an **X** to graph each point in the WAL27 table. Use a solid line to connect the points.

4. Use an **O** to graph each point in the NAL63 table. Use a dotted line to connect the points.



**Student Workbook: Do the Calculations—Graph Linear Equations (cont.)**

5. How many seconds will it take each plane to arrive at the point where the routes intersect?

WAL27 40 secondsNAL63 48 seconds

6. Will the planes meet at the point where the two routes intersect?

No

7. If No, which plane will arrive first?

WAL27

8. How many seconds will it take this plane to travel to the point where the two routes intersect?

40 seconds

9. At that time, how far away is the other airplane? (Hint: At that time, how many feet has the second plane traveled? How many feet is it from the intersection?)

Flight NAL63 is approximately 2.7 feet away.**After 40 seconds, Flight NAL63 has traveled approximately 13.3 feet along its 16-foot route. So NAL63 is approximately 2.7 feet from the intersection. So the planes do not meet.**

10. If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?

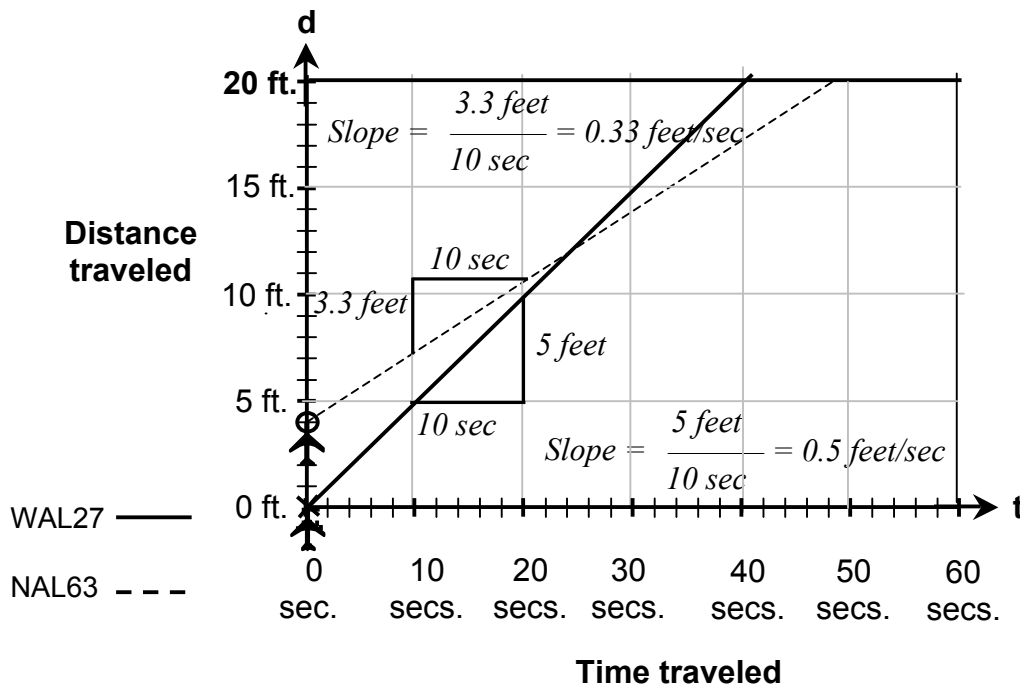
Change the speed or change the route of one of the planes.


Student Workbook: Do the Calculations—Graph Linear Equations (cont.)

Discussion: For each line, its slope represents the speed of the corresponding plane.

For the line corresponding to Flight WAL27, the slope of the line is 0.5 feet/sec, the speed of Flight WAL27.

For the line corresponding to Flight NAL63, the slope of the line is approximately 0.33 feet/sec, the speed of Flight NAL63.



11. Write the number that is the slope of the line representing WAL27. **1/2 feet/second**

12. Write the number that is the slope of the line representing NAL63. **1/3 feet/second**

13. What information does the slope of the line tell you about each plane?

For Flight WAL27, the slope of the line is 1/2 foot/second, the speed of Flight WAL27. For Flight NAL63, the slope of the line is 1/3 foot/second, the speed of Flight NAL63.

**Student Workbook: Analyze Your Results**

9. Are the plane speeds the same or different?

Different

10.

	Which plane is ahead?	Separation Distance (feet)
At the start:	Flight <u>NAL63</u>	Headstart = 4 feet
At the intersection:	Flight <u>WAL27</u>	Separation = <u>2.7</u> feet

Two planes are flying at different speeds on two different routes. The planes start at different distances from the point where the routes meet.

11. Do you have enough information to predict the separation distance between the planes at the point where the routes meet?

No

12. If NO, what other information do you need? **The plane speeds and the starting distance of each plane from the intersection.**



Student Workbook: Extension

1. Based upon your calculations, what is the difference in the planes' final positions?

(That is, what is the planes' separation distance where the routes meet?)

Approx. 2.7 feet

2. Does this distance satisfy the separation requirement?

No

3. If No, what would you tell the air traffic controller to do to meet the separation requirement?

Change the speed or change the route of one of the planes.

**Posttest**

1. Do you think the planes will meet at the point where the two routes intersect?

No

Why or why not? **Flight WAL27 travels 1 foot/second. So Flight WAL27 will travel 30 feet to the intersection in 30 seconds. Flight NAL63 travels 2/3 foot per second. So in 30 seconds, Flight NAL63 will travel 20 feet along its 24-foot route and will be 4 feet from the intersection.**

2. If not, how many feet apart do you think the planes will be when the first plane reaches the point where the routes intersect?

4 feet

3. Does your answer to Question 2 meet the 5-foot separation standard?

No

4. If you think two planes will *not* meet the 5-foot separation standard, what could you tell the air traffic controllers to do to make sure that the separation standard will be met? **Change the speed or change the route of one of the planes.**



Posttest (cont.)

Now consider this general problem.

Two planes are traveling at different speeds on two different routes.

The planes are different distances from the point where the two routes intersect.

5. Do you have enough information to predict
the separation distance between the planes
at the point where the routes come together?

No

If NO, what other information do you need? **The plane speeds and the starting distance of each plane from the intersection.**